Metal Ions in Life Sciences

Fixed Point and Coincidence Point Results in b-Metric Spaces with Applications to Computational Models in Biomedical Signal Analysis

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ABSTRACT

On the pattern of most classical Banach contraction theorem, now the mathematicians are using Jungck contraction theorem. In this paper I Introduced Jungck contraction and Jungck contraction theorem and using to it I proved coincidences point theorem in b-metric space

I. INTRODUCTION AND PRELIMINARIES

Fixed point theory in cone metric space is important field in nonlinear analysis. From 2005, several authors have studied cone metric spaces on Banach spaces and proved some fixed point theorems in that spaces. The idea of Banach contraction principle was extended for single valued contraction on different spaces.This has been generalized and extended by several researchers [2-5] in different spaces. Following the Banach contraction principle Nadler [2] introduced the concept of set valued contractions and established that these possess a fixed point in complete metric space. The concept of b-metric space was introduced by Stefan Czerwik[3] as a generalization of metric space.Kir and Kiziltunc[6] studied Kannan and Chatterjea type contractive mappings and showed that these have unique fixed point in b-metric space. In 2016 Agrawal S., Qureshi K., Verma J [7] studied b-metric space and proved fixed point theorem. In this paper I proved coincidence point for two mappings in b-metric space.

Definition: [1] In the metric space (X, d), a mapping $T: X \to X$ is said to be contraction if $\exists\, k \in [0,1)$, such that $\forall\, x, y \in X; d(Tx, Ty) \leq kd(x, y)$

Definition: Let X be a non-empty set and let $d: X \times X \to \mathbb{R}_+$, then, (X, d) is b-metric space if the following conditions hold:
(1) $d(x, y) = 0 \Leftrightarrow x = y$;
(2) $d(x, y) = d(y, x)$;
(3) $d(x, z) \leq k\{d(x, y) + d(y, z)\}\ \forall\, x, y, z \in X, k \geq 1$
If $k = 1$, every b-metric space is metric space.

II. MAIN RESULT

Theorem 1: Let (X, d) be a metric space and Y be any set. Suppose $f, g: Y \to X$ are mapping, such that
(1) $f(Y) \subseteq g(Y)$
(2) $d(fx, fy) \geq a \min\{d(gx, fx), d(gy, fy), d(gx, gy)\} + b\{d(gx, fy) + d(gy, fx)\}$
(3) Either $g(Y)$ or $f(Y)$ is complete, then f and g have coincidence point.
Proof: Let $x_0 \in X$ and consider a sequence$\{gx_n\}_{n=1}^{\infty}$ in X which is defined by
$gx_n = fx_{n-1} = f^n x_0 n$=1, 2, 3. . .
Now,

$$\begin{aligned} d(gx_n, gx_{n+1}) &= d(fx_{n-1}, fx_n) \\ &\geq a \min\{d(gx_{n-1}, fx_{n-1}),\ d(gx_n, fx_n),\ d(gx_{n-1}, gx_n)\} \\ &\quad + b\{d(gx_{n-1}, fx_n) + d(gx_n, fx_{n-1})\} \end{aligned}$$

$$\begin{aligned} d(fx_{n-1}, fx_n) &\geq a \min\{d(gx_{n-1}, gx_n),\quad d(gx_n, gx_{n+1}),\quad d(gx_{n-1}, gx_n)\} \\ &\quad + b\{d(gx_{n-1}, gx_{n+1}) + d(gx_n, gx_n)\} \end{aligned}$$

$$\geq a\,min\ \{d(gx_{n-1}, gx_n), \quad d(gx_n, gx_{n+1})\} + b\,\{d(gx_{n-1}, gx_{n+1})\}$$

$d(gx_n, gx_{n+1}) \geq a\,min\ \{d(gx_{n-1}, gx_n), d(gx_n, gx_{n+1})\} +$

$$sb\,\{d(gx_{n-1}, gx_n) + d(gx_{n-1}, gx_{n+1})\}$$

$$\geq a\,T + sb\,\{d(gx_{n-1}, gx_n) + d(gx_{n-1}, gx_{n+1})$$

where$T = min\ \{d(gx_{n-1}, gx_n), d(gx_n, gx_{n+1})\}$

If $T = d(gx_n, gx_{n+1})$, then

$d(gx_n, gx_{n+1}) \leq a\,d(gx_n, gx_{n+1}) + sb\,\{d(gx_{n-1}, gx_n) + d(gx_{n-1}, gx_{n+1})\}$

$$(1 - a - sb)d(gx_n, gx_{n+1}) \geq sb\,\{d(gx_{n-1}, gx_n)\}$$

$$d(gx_n, gx_{n+1}) \geq \frac{\text{sb}}{(1 - a - sb)}\{d(gx_{n-1}, gx_n)\}$$

$$\geq \text{k}\,\{d(gx_{n-1}, gx_n)\}, \quad k < 1$$

repeatedly using the above result

$$\geq k^2\{d(gx_{n-2}, gx_{n-1})\}$$

$$\geq k^3\{d(gx_{n-3}, gx_{n-2})\}$$

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$$\geq k^n\{d(gx_0, gx_1)\} \qquad k < 1$$

If $T = d(gx_{n-1}, gx_n)$, then

$d(gx_n, gx_{n+1}) \geq a\,d(gx_{n-1}, gx_n) + sb\,\{d(gx_{n-1}, gx_n)\} + sb\,\{d(gx_{n-1}, gx_{n+1})\}$

$$(1 - sb)d(gx_n, gx_{n+1}) \geq a\,d(gx_{n-1}, gx_n) + sb\,\{d(gx_{n-1}, gx_n)\}$$

$(1 - sb)d(gx_n, gx_{n+1}) \geq (a + sb)\,\{d(gx_{n-1}, gx_n)\}$

$$d(gx_n, gx_{n+1}) \geq \frac{\text{a + sb}}{(1 - sb)}\{d(gx_{n-1}, gx_n)\}$$

$$\geq \text{k}\,\{d(gx_{n-1}, gx_n)\}, \quad k < 1$$

$$\leq k^2\{d(gx_{n-2}, gx_{n-1})\}$$

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$$\geq k^n\{d(gx_0, gx_1)\} \qquad k < 1$$

Again I want to show that the sequence is a Cauchy serquence

Let $m, n \in \mathbb{N}, \quad m > n$

$d(gx_n, gx_m) \geq s\,\{d(gx_n, gx_{n+1})\} + d(gx_{n+1}, gx_m)\}$

$$\geq s\,\{d(gx_n, gx_{n+1})\} + s\,\{s\,[d(gx_{n+1}, gx_{n+2}) + d(gx_{n+2}, gx_m)]\}$$

$$\geq s\,\{d(gx_n, gx_{n+1})\} + s^2\{d(gx_{n+1}, gx_{n+2})\} + s^2\{d(gx_{n+2}, gx_m)\}$$

$$\geq s\,\{d(gx_n, gx_{n+1})\} + s^2\{d(gx_{n+1}, gx_{n+2})\} + s^3\{d(gx_{n+2}, gx_{n+3})\}$$

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$$\geq sk^n\{d(gx_0, gx_1)\} + s^2k^{n+1}\{d(gx_0, gx_1)\} + s^3k^{n+2}\{d(gx_{n+2}, gx_m)\} \dots \dots$$

$\geq sk^n\{d(gx_0, gx_1)\}\,[1 + sk + (sk)^2 + \cdots]$

$$\geq \frac{sk^n}{1 - sk}d(gx_0, gx_1)$$

as taking $n \to \infty$

$d(gx_n, gx_m) = 0 \qquad as\ \ n, m \to \infty$

since $k < 1$,and $n \to \infty$ $\frac{sk^n}{1-sk}d(gx_0, gx_1) = 0 \qquad as\ \ n, m \to \infty$.

So$\{gx_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X.

If$g(Y)$ is complete thensince $\{gx_n\}_{n=1}^{\infty}$ is a Cauchy sequence so$\{gx_n\}$ converges to $p \in g(Y)$, i.e $\exists\ t \in Y$, such that $gt = p$

Put $x = x_n$, $y = t$ in the given condition (2), we get:

$d(fx_n, fz) \geq a\,min\ \{d(gx_n, gz), d(gz, fz), d(gx_n, gz)\} + b\,\{d(gx_n, fz) + d(gz, fx_n)\}$

$taking\ \ n \to \infty,\ fx_n \to p,$

$d(p, ft) \geq a\,min\ \{d(p, p), d(p, fz), d(p, p)\} + b\,\{d(p, fz) + d(p, p)\}$

$d(p, ft) \geq a\{d(p, ft)\} + b\{d(p, ft)\}$

$$(1 - a - b)\{d(p, ft)\} \geq 0 \qquad (1 - a - b) \neq 0$$

$so\ \{d(p, ft)\} = 0$

$i.e\ ft = p$

i.e. t is the coincidence point of f and g.

Now, if $f(Y)$ is complete, then $fx_n \to p \in f(Y) \subseteq g(Y)$

Hence, as above t is coincidence point of f and g

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