

Nano $\alpha\psi$ -Closed Sets in Nano Topological Spaces: A Study in Generalized Topology with Applications to Biomedical Data Classification and Pattern Analysis

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ABSTRACT

In this article, we enclose the idea of nano alpha psi closed set in nano topological spaces and establish some of their properties. We also establish various forms of nano alpha psi locally closed, nano alpha psi locally closed star and nano alpha psi locally closed star star sets. Study some of related theorems.

Keywords: *Nan ψ -closed set, Nan ψ LC, Nan ψ LC* and Nan ψ LC**.*

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I. INTRODUCTION

O.Njastad [7] introduced the concept of α closed sets in topological spaces. M.K.R.S.Veerakumar[10] was introduced the notion of ψ closed sets. The notion $\alpha\psi$ closed set in topological spaces are introduced by R.Devi et.al. [2]. Lellis Thivagar [3] was introduced a new concept of nano topology, it was defined interms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He formulated the notion of nano closed set, nano interior and nano closure and also he introduced the notion of nano semi closed and nano α closed sets. The concept of nano generalized locally closed sets in nano topology were introduced by K.Bhuvaneswari et.al. [1].

In aim of this paper we introduce the concept of nano $\alpha\psi$ closed set and establish some of their properties. We also establish various forms of nano $\alpha\psi$ Locally closed, Nan ψ LC* and Nan ψ LC** sets. Study some of related theorems.

II. PRELIMINARIES

We recall the following definitions, which are useful in the sequel.

Definition 2.1. [3] Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Let X is a subset of U , then the lower approximation of X with respect to R is denoted by $\underline{R} = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.

Definition 2.2. [3] The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and its is denoted by $\overline{R} = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

Definition 2.3. [3] The boundary region of X with respect to R is the set of all objects, which can be possibly classified neither as X nor as not X with respect to R and its is denoted by $B_R = \overline{R} - \underline{R}$.

Definition 2.4. [3] If (U, R) is an approximation space and $X, Y \subseteq U$. Then

1. $\underline{R} \subseteq X \subseteq \overline{R}$
2. $\underline{R}(\emptyset) = \overline{R}(\emptyset) = \emptyset$ and $\underline{R}(U) = \overline{R}(U) = U$
3. $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$
4. $\overline{R}(X \cap Y) \subseteq \overline{R}(X) \cap \overline{R}(Y)$

5. $\overline{R}(X \cup Y) \supseteq \overline{R}(X) \cup \overline{R}(Y)$
6. $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$
7. $\overline{R}(X) \subseteq \overline{R}(Y)$ and $\underline{R}(X) \subseteq \underline{R}(Y)$ whenever $X \subseteq Y$
8. $\overline{R}(X^c) = (\underline{R})^c$ and $\underline{R}(X^c) = (\overline{R})^c$
9. $\underline{R}(\underline{R}(X)) = \overline{R}(\overline{R}(X)) = \underline{R}(X)$
10. $\overline{R}(\overline{R}(X)) = \underline{R}(\underline{R}(X)) = \overline{R}(X)$
11. $R(X \cap Y) \subseteq R(X) \cap R(Y)$

Definition 2.5. [3] Let U be an universe and R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, \overline{R}, \underline{R}, B_R\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.
2. The union of the element of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the element of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U is called the nano topology on U with respect to X . $(U, \tau_R(X))$ as the nano topological space. The element of $\tau_R(X)$ are called as nano open sets and complement of nano open sets is called nano closed.

Definition 2.6. [3] If $(U, \tau_R(X))$ is a nano topological spaces with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano open subsets contained in A and it is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest nano open subsets contained in A and is defined as the intersection of all nano closed sets containing A and it is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest nano closed set containing A .

III. NANO $\alpha\psi$ -CLOSED SETS

Definition 3.1. A subset H of $(U, \tau_R(X))$ is called nano $\alpha\psi$ -closed set if $\psi cl(H) \subseteq V$ whenever $H \subseteq V$ and V is nano α -open in $(U, \tau_R(X))$. The complement of nano $\alpha\psi$ -closed set is nano $\alpha\psi$ -open set.

Theorem 3.2. Each nano closed set is a nano $\alpha\psi$ -closed set.

Proof. Let H be a nano α open set in $(U, \tau_R(X))$ and H be a nano closed set in nano topological spaces $(U, \tau_R(X))$ then $A = Ncl(A)$. Every nano closed set is nano ψ closed set. Hence $N\psi cl(H) \subseteq Ncl(H) = H$. Therefore $N\psi cl(H) \subseteq H$. Hence H is $N\alpha\psi$ closed set.

The nano $\alpha\psi$ -closed set is not nano closed set, this is proved by the following example.

Example 3.3. Let $U = \{1, m, n, o\}$ with $U/R = \{\{1\}, \{n\}, \{m, o\}\}$ and $X = \{1, m\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{1\}, \{1, m, o\}, \{m, o\}\}$. Here $N\alpha\psi Cl = \{U, \phi, \{1\}, \{m\}, \{n\}, \{o\}, \{1, n\}, \{m, n\}, \{n, o\}, \{m, o\}, \{1, m, n\}, \{m, n, o\}, \{1, n, o\}\}$ and let $A = \{1\}$. Then A is not nano closed but it is nano $\alpha\psi$ -closed set.

Theorem 3.4. Each nano closed set is a nano ψ -closed set.

Proof. Let H be a nano ψ open set in $(U, \tau_R(X))$, and H be a nano closed set in nano topological spaces $(U, \tau_R(X))$ then $H = Ncl(H)$. Every nano closed set is nano semi closed set. Hence $Nscl(H) \subseteq Ncl(H) = H$. Therefore $Nscl(H) \subseteq H$. Hence H is $N\psi$ closed set. The nano ψ -closed set is not nano closed set, this is proved by the following example.

Example 3.5. Let $U = \{1, m, n, o\}$ with $U/R = \{\{1\}, \{n\}, \{m, o\}\}$ and $X = \{1, m\}$. Then the nano topology $\tau_R(X) = \{U, \phi, \{1\}, \{1, m, o\}, \{m, o\}\}$. Here $N\psi Cl = \{U, \phi, \{1\}, \{n\}, \{1, n\}, \{m, o\}, \{m, n, o\}\}$ and let $A = \{m, n\}$. Then A is not nano closed but it is nano ψ -closed set.

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Theorem 3.6. Each nano α closed set is a nano $\alpha\psi$ -closed set.

Proof. Let H be a nano α open set in $(U, \tau_R(X))$, then $H = N\alpha cl(H)$. Suppose $H \subseteq V$, V is $N\alpha$ open set. Since H is $N\alpha$ closed, $N\psi cl(H) \subseteq N\alpha cl(H) \subseteq V$. Therefore H is $N\alpha\psi$ closed set. The nano $\alpha\psi$ -closed set is not nano α closed set, this is proved by the following example.

Example 3.7. Let $U = \{l, m, n, o\}$ with $U/R = \{\{l\}, \{n\}, \{m, o\}\}$ and $X = \{l, m\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{l\}, \{l, m, o\}, \{m, o\}\}$. Here $N\alpha\psi Cl = \{U, \emptyset, \{l\}, \{m\}, \{n\}, \{o\}, \{l, n\}, \{m, n\}, \{n, o\}, \{m, o\}, \{l, m, n\}, \{m, n, o\}, \{l, n, o\}\}$ and let $A = \{n, o\}$. Then A is not nano α closed but it is nano $\alpha\psi$ -closed set.

Theorem 3.8. Each nano semi closed set is a nano $\alpha\psi$ -closed set.

Proof. Let H be a nano semi closed set in $(U, \tau_R(X))$, then $H = Nscl(H)$. Suppose $H \subseteq V$, V is $N\alpha$ open set. Since H is nano semi closed, $N\psi cl(H) \subseteq Nscl(H) \subseteq V$. Therefore H is $N\alpha\psi$ closed set. The nano $\alpha\psi$ -closed set is not nano semi closed set, this is proved by the following example.

Example 3.9. Let $U = \{l, m, n, o\}$ with $U/R = \{\{l\}, \{n\}, \{m, o\}\}$ and $X = \{l, m\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{l\}, \{l, m, o\}, \{m, o\}\}$. Here $N\alpha\psi Cl = \{U, \emptyset, \{l\}, \{m\}, \{n\}, \{o\}, \{l, n\}, \{m, n\}, \{n, o\}, \{m, o\}, \{l, m, n\}, \{m, n, o\}, \{l, n, o\}\}$ and let $A = \{l, m, n\}$. Then A is not nano semi closed but it is nano $\alpha\psi$ -closed set.

Theorem 3.10. Each nano sg closed set is a nano $\alpha\psi$ -closed set.

Proof. Let H be a nano sg closed set in $(U, \tau_R(X))$. Suppose $H \subseteq V$, V is $N\alpha$ open set. Every $N\alpha$ open set is N semi open. Since H is nano semi closed, $Nscl(H) \subseteq V$ then $N\psi cl(H) \subseteq Nscl(H) \subseteq V$. Therefore H is $N\alpha\psi$ closed set. The nano $\alpha\psi$ -closed set is not nano sg closed set, this is proved by the following example.

Example 3.11. Let $U = \{l, m, n, o\}$ with $U/R = \{\{l, m\}, \{n\}, \{o\}\}$ and $X = \{l, n\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{n\}, \{l, m, n\}, \{l, m\}\}$. Here $N\alpha\psi Cl$ is power set and let $A = \{l, m, n\}$. Then A is not nano sg closed but it is nano $\alpha\psi$ -closed set.

Theorem 3.12. Each nano $g\alpha$ closed set is a nano $\alpha\psi$ -closed set.

Proof. Let H be a nano $g\alpha$ closed set in $(U, \tau_R(X))$. Suppose $H \subseteq V$, V is $N\alpha$ open set. Since $N\alpha cl(H) \subseteq V$ then $N\psi cl(H) \subseteq N\alpha cl(H) \subseteq V$. Therefore H is $N\alpha\psi$ closed set. The nano $\alpha\psi$ -closed set is not nano $g\alpha$ closed set, this is proved by the following example.

Example 3.13. Let $U = \{l, m, n, o\}$ with $U/R = \{\{l\}, \{n\}, \{m, o\}\}$ and $X = \{l, m\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{l\}, \{l, m, o\}, \{m, o\}\}$. Here $N\alpha\psi Cl = \{U, \emptyset, \{l\}, \{m\}, \{n\}, \{o\}, \{l, n\}, \{m, n\}, \{n, o\}, \{m, o\}, \{l, m, n\}, \{m, n, o\}, \{l, n, o\}\}$ and let $A = \{l, m, n\}$. Then A is not nano $g\alpha$ closed but it is nano $\alpha\psi$ -closed set.

Theorem 3.14. Each nano ψ closed set is a nano $\alpha\psi$ -closed set.

Proof. Let H be a nano ψ closed set in $(U, \tau_R(X))$. Suppose $H \subseteq V$, V is $N\alpha$ open set. Every nano α open is nano semi open and every nano semi open set is nano sg open. Since $Nscl(H) \subseteq V$ then $N\psi cl(H) \subseteq Nscl(H) \subseteq V$. Therefore H is $N\alpha\psi$ closed set.

The nano $\alpha\psi$ -closed set is not nano ψ closed set, this is proved by the following example.

Example 3.15. Let $U = \{l, m, n, o\}$ with $U/R = \{\{l\}, \{n\}, \{m, o\}\}$ and $X = \{l, m\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{l\}, \{l, m, o\}, \{m, o\}\}$. Here $N\alpha\psi Cl = \{U, \emptyset, \{l\}, \{m\}, \{n\}, \{o\}, \{l, n\}, \{m, n\}, \{n, o\}, \{m, o\}, \{l, m, n\}, \{m, n, o\}, \{l, n, o\}\}$ and let $A = \{m, n\}$. Then A is not nano ψ closed but it is nano $\alpha\psi$ -closed set.

Theorem 3.16. Union of two nano $\alpha\psi$ -closed sets are nano $\alpha\psi$ -closed set.

Proof. Let F and G be two nano $\alpha\psi$ -closed sets, Let E be any $N\alpha$ open set in $(U, \tau_R(X))$, such that $F \cup G \subseteq E$. Then $F \subseteq E$ and $G \subseteq E$. Since F and G are nano $\alpha\psi$ -closed set, $N\psi cl(F) \subseteq E$ and $N\psi cl(G) \subseteq E$. Therefore $N\psi cl(F) \cup N\psi cl(G) = N\psi cl(F \cup G) \subseteq E$. Hence $F \cup G$ is $N\alpha\psi$ -closed set.

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Theorem 3.17. Let H be a subset and X is $N\alpha\psi$ -closed set in $(U, \tau_R(X))$ then $N\psi\text{cl}(H) - H$ does not contain any empty $N\alpha$ closed sets in $(U, \tau_R(X))$.

Proof. Let H is $N\alpha\psi$ closed sets and G be a non empty $N\alpha$ closed subset of $N\psi\text{cl}(H) - H$. Then $G \subseteq N\psi\text{cl}(H) \cap (X - H)$. Since $X - H$ is $N\alpha$ open and H is $N\alpha\psi$ closed set. $N\psi\text{cl}(H) \subseteq X - H$ therefore $G \subseteq X - N\psi\text{cl}(H)$. Thus $G \subseteq N\psi\text{cl}(H) \cap (X - N\psi\text{cl}(H)) = \emptyset$. This implies that $G = \emptyset$. Thus $N\psi\text{cl}(H) - H$ does not contain any non empty $N\alpha\psi$ closed.

Theorem 3.18. If F is $N\alpha\psi$ -closed set in U and $F \subseteq G \subseteq N\alpha\psi\text{cl}(F)$ then G is also $N\alpha\psi$ -closed set in U . **Proof.** Suppose F is $N\alpha\psi$ -closed set in U . Let $G \subseteq U$ such that V is $N\alpha$ open set in U . Since $F \subseteq G$, we have $F \subseteq V$. Since F is $N\alpha\psi$ closed and $N\psi\text{cl}(G) \subseteq N\psi\text{cl}(N\psi\text{cl}(F)) = N\psi\text{cl}(F) \subseteq V$. Therefore $N\psi\text{cl}(G) \subseteq V$. Hence G is $N\alpha\psi$ closed set in U .

The inverse of the above theorem need not true be true by the following example.

Example 3.19. Let $U = \{l, m, n, o\}$ with $U/R = \{\{l\}, \{n\}, \{m, o\}\}$ and $X = \{l, m\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{l\}, \{l, m, o\}, \{m, o\}\}$. Here $N\alpha\psi\text{Cl} = \{U, \emptyset, \{l\}, \{m\}, \{n\}, \{o\}, \{l, n\}, \{m, n\}, \{n, o\}, \{m, o\}, \{l, m, n\}, \{m, n, o\}, \{l, n, o\}\}$ and the set $F = \{l\}$ and $G = \{l, n\}$. Such that F and G are $N\alpha\psi$ -closed sets but $F \subseteq G$ but G is not a subset of $N\alpha\psi\text{cl}(F)$.

Theorem 3.20. Let H is $N\alpha$ -open and $N\alpha\psi$ -closed set then H is $N\psi$ closed.

Proof. Since $H \subseteq H$ and H is $N\alpha$ open and $N\alpha\psi$ closed, we have $N\psi\text{cl}(H) \subseteq H$. Thus $N\psi\text{cl}(H) = H$. Hence H is $N\psi$ closed set in U .

Theorem 3.21. A set H is $N\alpha\psi$ -open in $(U, \tau_R(X))$ iff $F \subseteq N\psi\text{int}(H)$ whenever F is $N\alpha$ closed in $(U, \tau_R(X))$ and $F \subseteq H$.

Proof. Suppose $F \subseteq N\psi\text{int}(H)$ where F is $N\alpha$ -closed and $F \subseteq H$. Let $X - H \subseteq G$ where G is $N\alpha$ open in $(U, \tau_R(X))$. Then $G \subseteq X - G$ and $X - G \subseteq N\psi\text{int}(H)$. Thus $X - H$ is $N\alpha\psi$ -closed set in $(U, \tau_R(X))$. Hence H is $N\alpha\psi$ -open in $(U, \tau_R(X))$. Inversely, Suppose that H is $N\alpha\psi$ -open in $(U, \tau_R(X))$. $F \subseteq H$ and F is $N\alpha$ closed in $(U, \tau_R(X))$. Then $X - F$ is $N\alpha$ -open and $X - H \subseteq X - F$. Therefore $N\psi\text{cl}(X - H) \subseteq X - F$. But $N\psi\text{cl}(X - H) = X - N\psi\text{int}(H)$. Hence $F \subseteq N\psi\text{int}(H)$.

Theorem 3.22. A subset H is $N\alpha\psi$ -open in $(U, \tau_R(X))$ iff $G = X$ whenever G is $N\alpha$ open and $N\psi\text{int}(H) \cup (X - G) \subseteq G$.

Proof. Let H be $N\alpha\psi$ -open. G be $N\alpha$ -open and $N\psi\text{int}(H) \cup (X - H) \subseteq G$. This gives $X - G \subseteq (X - \psi\text{int}(H)) \cap (X - (X - H)) = X - N\psi\text{int}(H) - (X - H) = N\psi\text{cl}(X - H) - (X - H)$. Since $X - H$ is $N\alpha\psi$ -closed and $X - G$ is $N\alpha$ -closed. Then by Theorem 3.21 it follows that $X - G = \emptyset$. Therefore $X = G$. Inversely, Suppose F is $N\alpha$ -closed and $F \subseteq H$. Then $N\psi\text{int}(H) \cup (X - H) \subseteq (H) \cup (X - F)$. It follows that $\psi\text{int}(H) \cup (X - F) = X$ and hence $F \subseteq N\psi\text{int}(H)$. Therefore H is $N\alpha\psi$ -open in $(U, \tau_R(X))$.

IV. NANO $\alpha\psi$ -LOCALLY CLOSED SETS

We introduce the following definition.

Definition 4.1. Let H be a subset of nano topological spaces is called nano $\alpha\psi$ locally closed sets ($N\alpha\psi\text{LC}$) if $H = K \cap R$, where K is nano $\alpha\psi$ open and R is nano $\alpha\psi$ closed sets in nano topological spaces.

Definition 4.2. Let H be a subset of nano topological spaces is called nano $\alpha\psi$ locally closed * sets ($N\alpha\psi\text{LC}^*$) if $H = K \cap R$, where K is nano $\alpha\psi$ open and R is nano closed sets in nano topological spaces.

Definition 4.3. Let H be a subset of nano topological spaces is called nano $\alpha\psi$ locally closed sets ($N\alpha\psi\text{LC}^{**}$) if $H = K \cap R$, where K is nano open and R is nano $\alpha\psi$ closed sets in nano topological spaces.

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Example 4.4. Let $U = \{l, m, n, o\}$ with $U/R = \{\{l\}, \{n\}, \{m, o\}\}$ and $X = \{l, m\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{l\}, \{l, m, o\}, \{m, o\}\}$. Here $N\alpha\psi Cl = \{U, \emptyset, \{l\}, \{m\}, \{n\}, \{o\}, \{l, n\}, \{m, n\}, \{n, o\}, \{m, o\}, \{l, m, n\}, \{m, n, o\}, \{l, n, o\}\}$. Then $NLC = \{U, \emptyset, \{l\}, \{n\}, \{l, n\}, \{m, o\}, \{l, m, o\}, \{m, n, o\}\}$ and 1. $N\alpha\psi LC$ is power set of U . 2. $N\alpha\psi LC^* = \{U, \emptyset, \{l\}, \{m\}, \{n\}, \{o\}, \{l, n\}, \{l, m\}, \{l, o\}, \{m, o\}, \{m, n, o\}, \{l, m, o\}\}$. 3. $N\alpha\psi LC^{**} = \{U, \emptyset, \{l\}, \{m\}, \{n\}, \{o\}, \{l, n\}, \{m, n\}, \{n, o\}, \{m, o\}, \{l, m, n\}, \{m, n, o\}, \{l, n, o\}, \{l, m, o\}\}$

Remarks 4.5.

1. Let H be a subset of NT , is nano $\alpha\psi$ locally closed iff $(U - H)^c$ is equal to union of all $\tau_R(x)$ and $\tau^c R(x)$.
2. Each nano $\alpha\psi$ open subset of U is nano $\alpha\psi$ locally closed sets and each nano $\alpha\psi$ closed subset of U is nano $\alpha\psi$ locally closed sets.
3. $[N\alpha\psi LC]^c$ is need not be a nano $\alpha\psi$ locally closed sets.

Theorem 4.6. Each nano $\alpha\psi$ closed set is nano $\alpha\psi$ locally closed sets.

Proof. Obviously true by definition.

Theorem 4.7. A subset H of NT , 1. If H is nano locally closed, then $H \in N\alpha\psi LC$ ($H \in N\alpha\psi LC^*$ and $H \in N\alpha\psi LC^{**}$) but inverse need not be true. 2. If $H \in N\alpha\psi LC^*$ or $H \in N\alpha\psi LC^{**}$, then H is nano $\alpha\psi$ locally closed.

Proof. This is proved by the following example.

Example 4.9.

1. Here all NLC set is $N\alpha\psi LC$ because it is power set of U , it is also $N\alpha\psi LC^*$ and $N\alpha\psi LC^{**}$. It is proved obviously by Example [4.4].
2. This is also proved by Example [4.4] that is Each $N\alpha\psi LC^*$ or $N\alpha\psi LC^{**}$ is $N\alpha\psi LC$ set because it is power set of U .

Theorem 4.10. Let H be a subset of nano topology, then the following conditions are equivalent.

1. A subset H belongs to $N\alpha\psi LC^*$.
2. A subset H is equal to the intersection of both $N\alpha\psi$ open set K and nano closure of H .
3. $Ncl(H) - H$ is nano $\alpha\psi$ closed.
4. Union of all H and U difference nano closure of H is nano $\alpha\psi$ open.

Proof. (1) \Rightarrow (2) Suppose a subset H belongs to $N\alpha\psi LC^*$ in nano topology. Then $H = K \cap R$ where K is $N\alpha\psi$ open and R is nano closed. Since H is a subset of K and $H \subseteq Ncl(H)$, $H \subseteq K \cap Ncl(H)$. Inversely, let H is a subset of R , $Ncl(H) \subseteq R$, we have $H = K \cap R$ contains $K \cap Ncl(H)$. Therefore $K \cap Ncl(H) \subseteq H$. Hence H is equal to the intersection of both $N\alpha\psi$ open set K and nano closure of H .

(2) \Rightarrow (1) Let K is $N\alpha\psi$ open and $Ncl(H)$ is $N\alpha\psi$ closed $K \cap Ncl(H)$ belongs to $N\alpha\psi LC^*$.

(2) \Rightarrow (3) Let a subset H is equal to the intersection of both $N\alpha\psi$ open set K and nano closure of H implies that $Ncl(H) - H = Ncl(H) \cap K^c$ which is $N\alpha\psi$ closed, since K^c is $N\alpha\psi$ closed.

(3) \Rightarrow (2) Suppose $K = [Ncl(H) - H]^c$, then by assumption K is $N\alpha\psi$ open in NT and $H = K \cap Ncl(H)$.

(3) \Rightarrow (4) $H \cup (U - Ncl(H)) = H \cup (Ncl(H))^c = (Ncl(H) - H)^c$ and by (3) $(Ncl(H) - H)^c$ is $N\alpha\psi$ open and $H \cup (U - Ncl(H))$ is $N\alpha\psi$ open.

(4) \Rightarrow (3) Suppose $K = H \cup [Ncl(H)]^c$. Then K^c is $N\alpha\psi$ closed and $K^c = Ncl(H) - H$ and hence $Ncl(H) - H$ is $N\alpha\psi$ closed.

Theorem 4.11. Let H be a subset of nano topology, then the following conditions are equivalent.

1. A subset H belongs to $N\alpha\psi LC$ set in nano topology.
2. Let H be a subset and which is equal to the intersection of both K for some $N\alpha\psi$ open set and nano $\alpha\psi$ closure of H .
3. $N\alpha\psi cl(H) - H$ is nano $\alpha\psi$ closed.
4. Union of all H and $N\alpha\psi cl(H)^c$ is nano $\alpha\psi$ open.
5. H is a subset of $N\alpha\psi int(H \cup (N\alpha\psi cl(H))^c)$.

Proof. (1) \Rightarrow (2) Let H belongs to $N\alpha\psi LC$ in nano topology. Then $H = K \cap R$ where K is $N\alpha\psi$ open and R is $N\alpha\psi$ closed. Since H is a subset of K and $H \subseteq N\alpha\psi cl(H)$ implies $H \subseteq K \cap N\alpha\psi cl(H)$. Inversely, let H is a subset of R ,

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$N\alpha\psi\text{cl}(H) \subseteq R$ and hence $K \cap N\alpha\psi\text{cl}(H) \subseteq H$. Therefore $K \cap N\alpha\psi\text{cl}(H) \subseteq H$. Hence H is equal to the intersection of both $N\alpha\psi$ open set K and nano closure of H .

(2) \Rightarrow (3) $H = K \cap N\alpha\psi\text{cl}(H) \Rightarrow N\alpha\psi\text{cl}(H) - H = N\alpha\psi\text{cl}(H) \cap K^c$ which is $N\alpha\psi$ closed.

(3) \Rightarrow (4) $H \cap (N\alpha\psi\text{cl}(H))^c = (N\alpha\psi\text{cl}(H) - H)^c$ and by assumption $(N\alpha\psi\text{cl}(H) - H)^c$ is $N\alpha\psi$ open and it is $H \cap (N\alpha\psi\text{cl}(H))^c$.

(4) \Rightarrow (5) By assumption, $H \cup (N\alpha\psi\text{cl}(H))^c = N\alpha\psi\text{int}(H \cup (N\alpha\psi\text{cl}(H))^c)$ and hence $H \subseteq N\alpha\psi\text{int}(H \cap (N\alpha\psi\text{cl}(H))^c)$. (5)

\Rightarrow (1) By assumption, $H \subseteq N\alpha\psi\text{cl}(H)$, $H = N\alpha\psi\text{int}(H \cup N\alpha\psi\text{cl}(H))^c \cap N\alpha\psi\text{cl}(H) \in N\alpha\psi\text{LC}$ in NT.

Theorem 4.12. A subset H of nano topology, then H belongs to $N\alpha\psi\text{LC}^*$ iff $H = K \cap N\alpha\psi\text{cl}(H)$ for some nano open set K . Proof. Suppose H belongs to $N\alpha\psi\text{LC}^{**}$ in nano topology. Then $H = K \cap R$ where K is nano open and R is $N\alpha\psi$ closed. Since, $H \subseteq R$, $N\alpha\psi\text{cl}(H) \subseteq R$. Now $H = H \cap N\alpha\psi\text{cl}(H) = K \cap R \cap N\alpha\psi\text{cl}(H) = K \cap N\alpha\psi\text{cl}(H)$. Here the inverse part is true by definition.

Theorem 4.13. A subset H of nano topology. If H belongs to $N\alpha\psi\text{LC}^{**}$ in nano topology, then $N\alpha\psi\text{cl}(H) - H$ is $N\alpha\psi$ closed and $H \cup (N\alpha\psi\text{cl}(H))^c$ is $N\alpha\psi$ open.

Proof. Suppose that H belongs to $N\alpha\psi\text{LC}^{**}$ in nano topology. Then by Theorem [4.12], $H = K \cap N\alpha\psi\text{cl}(H)$ for some nano open K and $N\alpha\psi\text{cl}(H) - H = N\alpha\psi\text{cl}(H) \cap K^c$ is $N\alpha\psi$ closed. If $R = N\alpha\psi\text{cl}(H) - H$ then $R^c = H \cup (N\alpha\psi\text{cl}(H))^c$ and R^c is $N\alpha\psi$ open and hence $H \cup (N\alpha\psi\text{cl}(H))^c$ is $N\alpha\psi$ open.

V. CONCLUSION

In this paper we have discussed about the new concept of nano closed set in nano topological spaces. We learn about the relationship between this set and already existing sets. Further we study the concept of nano $\alpha\psi$ Locally closed, $N\alpha\psi\text{LC}^*$ and $N\alpha\psi\text{LC}^{**}$ sets and also derive some of their related properties.

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